

# Hydrodynamics of an ultra-relativistic fluid in the flat anisotropic cosmological model

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Motion of an ultra-relativistic perfect fluid in space-time with the Kasner metrics is investigated by the Hamiltonian method. It is found that in the limit of small times a tendency takes place to formation of strong inhomogeneities in matter distribution. In the case of slow flows the effect of non-stationary anisotropy on dynamics of sound waves and behaviour of frozen-in vortices is considered. It is shown that hydrodynamics of slow vortices on the static homogeneous background is equivalent to the usual Eulerian incompressible hydrodynamics, but in the presence of an external non-stationary strain velocity field.

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Dynamics of a relativistic fluid in a gravitational field has a set of distinctive properties in comparison with hydrodynamics in a flat space-time [1], [2]. An explicit dependence of the metric tensor components  $g_{ik}(t, \mathbf{r})$  on time coordinate  $t$  and spatial coordinates  $\mathbf{r} = (x, y, z)$ , together with back influence of the matter on gravitational field, result in a wide variety in behaviour of solutions for equations of motion. Many phenomena in the general-relativistic hydrodynamics have not been studied yet. In order to get a better understanding of various processes taking place in the system, it is useful to analyse simple limit cases, when only one of many general-relativistic effects is mostly important. If a dissipation may be neglected, then one can simplify significantly the analysis by using the Hamiltonian formalism, application of which to hydrodynamics has given many interesting results (see the reviews [3], [4] and references therein). A brief discussion of adaptation of the Hamiltonian method for the relativistic hydrodynamics can be found, for instance, in the papers [5] and [6]. So, in the work [6], slow isentropic flows of an ideal relativistic fluid have been considered and a partial study of influence of a spatial inhomogeneity on the motion of hydrodynamical vortices has been performed. On the contrary, in present work we suppose that the space is homogeneous and concentrate our attention on the effects of its non-stationary anisotropy. It is known that there exists the class of solutions of the Einstein equations for empty space-time, which has been found by Kasner in 1922, with metrics of the following form [2]:

$$ds^2 = dt^2 - t^{2/3}(t^{2\lambda_1}dx^2 + t^{2\lambda_2}dy^2 + t^{2\lambda_3}dz^2), \quad (1)$$

where  $\lambda_1, \lambda_2, \lambda_3$  are any three numbers satisfying two conditions

$$\lambda_1 + \lambda_2 + \lambda_3 = 0, \quad \lambda_1^2 + \lambda_2^2 + \lambda_3^2 = 2/3. \quad (2)$$

At small times, this metrics describes approximately also the case of Universe filled uniformly by a fluid, since in the limit  $t \rightarrow 0$  a back reaction of matter on the gravitational field appears to be negligible [2], [7]. In present work we have used this result and considered the hydro-

dynamics of a perfect fluid having ultra-relativistic equation of state, in the space-time with the fixed metrics (1). We have established that at early times, flows with a finite mean momentum are in such a dynamic regime that a motion of each fluid element is nearly independent on the motion of other fluid elements. Due to this reason, there exists a tendency to formation of strong inhomogeneities in the spatial distribution of the matter which is related to singularities of an approximate Lagrangian mapping. We have found also that even in the case of slow flows with zero mean momentum the nonstationary anisotropy influences strongly both on the dynamics of sound oscillations and on the vortex motion, making their properties be different in some aspects from properties of the analogous objects in the flat space-time.

The plan of further exposition is the following. At first, a necessary explanation will be given concerning the employed theoretical method, and the expression for the Lagrangian  $\mathcal{L}$  of relativistic ideal hydrodynamics in a given gravitational field will be written. After a brief discussion of specific properties of corresponding equations of motion, we shall pass to the Hamiltonian description of the system dynamics and we will introduce the exact expression for the Hamiltonian  $\mathcal{H}$  corresponding to the ultra-relativistic equation of state of the matter. Some important properties of the dynamics in the limit  $t \rightarrow 0$  will be discussed. A quadratic (on small deviations) part  $\mathcal{H}^{(2)}$  of the Hamiltonian will be used for the derivation of simplified equations of motion for sound waves and vortices.

At first we introduce needed concepts. The energy density  $\varepsilon$ , measured in the locally co-moving reference frame, and the density  $n$  of number of conserved particles are connected by equation of state  $\varepsilon = \varepsilon(n)$  [1]. The scalar  $n$  is equal to the length of the 4-vector of current  $n^i = n(dx^i/ds)$  [2]. Let us introduce the relative density  $\rho(t, \mathbf{r})$  in such a manner that it obeys the standard continuity equation

$$\rho_t + \nabla(\rho \mathbf{v}) = 0, \quad (3)$$

where 3-velocity field  $\mathbf{v}(t, \mathbf{r}) = v^\alpha = (v^x, v^y, v^z)$  is defined as  $dx^\alpha/dt$  on the world-line of the fluid point pass-

ing through  $(t, \mathbf{r})$ . The equation (3) follows from the equation  $n_{,i}^i = 0$ , expressing the conservation law for the fluid amount [1], if  $\rho$  and  $n$  are connected by the relation [6]:

$$n = \frac{\rho}{\sqrt{|g|}} \sqrt{g_{00} + 2g_{0\alpha}v^\alpha + g_{\alpha\beta}v^\alpha v^\beta}, \quad (4)$$

where  $g = \det\|g_{ik}\|$  is the determinant of the metric tensor.

The action functional  $S = \int \mathcal{L} dt$  of relativistic hydrodynamics is defined through the Lagrangian  $\mathcal{L} = \mathcal{L}\{\rho, \mathbf{v}\}$  in the following way [5], [6]:

$$\mathcal{L} = - \int \varepsilon \left( \frac{\rho}{\sqrt{|g|}} \sqrt{g_{00} + 2g_{0\alpha}v^\alpha + g_{\alpha\beta}v^\alpha v^\beta} \right) \sqrt{|g|} d\mathbf{r}. \quad (5)$$

The equation of motion for the velocity field  $\mathbf{v}(t, \mathbf{r})$  has the structure (generalized Euler's equation [8])

$$(\partial_t + \mathbf{v} \cdot \nabla) \left( \frac{1}{\rho} \cdot \frac{\delta \mathcal{L}}{\delta \mathbf{v}} \right) = \nabla \left( \frac{\delta \mathcal{L}}{\delta \rho} \right) - \frac{1}{\rho} \left( \frac{\delta \mathcal{L}}{\delta v^\alpha} \right) \nabla v^\alpha. \quad (6)$$

This is merely the variational Euler-Lagrange equation expressing the least action  $S$  principle for variations of world-lines of fluid particles. The 3-vector  $\mathbf{p}(t, \mathbf{r}) = (1/\rho)(\delta \mathcal{L}/\delta \mathbf{v})$ ,

$$p_\alpha = \varepsilon'(n) \frac{-(g_{0\alpha} + g_{\alpha\beta}v^\beta)}{\sqrt{g_{00} + 2g_{0\alpha}v^\alpha + g_{\alpha\beta}v^\alpha v^\beta}}, \quad (7)$$

is the canonical momentum of a liquid particle in the point  $(t, \mathbf{r})$ . We see that the definition of the canonical momentum in relativistic hydrodynamics depends in a complicated manner on the equation of state. The relation (7) can be resolved respectively to the velocity, which gives the inverse dependence:  $\mathbf{v} = \mathbf{v}\{\rho, \mathbf{p}\}$ . The remarkable equation for the vorticity field  $\mathbf{\Omega}(\mathbf{r}, t) = \text{curl } \mathbf{p}(t, \mathbf{r})$

$$\mathbf{\Omega}_t = \text{curl}[\mathbf{v} \times \mathbf{\Omega}], \quad (8)$$

can be obtained from the equation (6) by applying the curl-operator. It implies that vorticity is frozen in the fluid and the generalized Kelvin's theorem (conservation of circulation of the field  $\mathbf{p}$  along arbitrary closed contour  $\gamma(t)$  transported by flow) holds:

$$\oint_{\gamma(t)} (\mathbf{p} \cdot d\mathbf{l}) = \Gamma_\gamma = \text{const.}$$

Passing to representation of flows in terms of the fields  $\rho$  and  $\mathbf{p}$  and defining the Hamiltonian  $\mathcal{H}\{\rho, \mathbf{p}\}$  in such a manner:

$$\mathcal{H}\{\rho, \mathbf{p}\} = \left( \int \left( \frac{\delta \mathcal{L}}{\delta \mathbf{v}} \cdot \mathbf{v} \right) d\mathbf{r} - \mathcal{L} \right) \Big|_{\mathbf{v}=\mathbf{v}\{\rho, \mathbf{p}\}} \quad (9)$$

we have the equations of motion in the form [8]

$$\rho_t + \nabla \left( \frac{\delta \mathcal{H}}{\delta \mathbf{p}} \right) = 0, \quad (10)$$

$$\mathbf{p}_t = \left[ \left( \frac{\delta \mathcal{H}}{\delta \mathbf{p}} \right) \times \frac{\text{curl } \mathbf{p}}{\rho} \right] - \nabla \left( \frac{\delta \mathcal{H}}{\delta \rho} \right). \quad (11)$$

In particular, for the potential flows, when the vorticity is equal to zero and  $\mathbf{p} = \nabla \varphi$ , the dynamical variables  $\rho$  and  $\varphi$  are canonically conjugated:

$$\rho_t = \frac{\delta \mathcal{H}\{\rho, \nabla \varphi\}}{\delta \varphi}, \quad -\varphi_t = \frac{\delta \mathcal{H}\{\rho, \nabla \varphi\}}{\delta \rho}. \quad (12)$$

Equations (12) will be used further (in linearized form) for analysis of acoustic mode dynamics.

Other interesting dynamic regime can take place if there is an equilibrium solution  $\rho = \rho_0(\mathbf{r})$ ,  $\mathbf{p} = \mathbf{p}_0 = \text{const}$ , independent on time. If vortical component of flow is small and the sound waves are excited weakly, then perturbations of density are negligibly small in comparison with its equilibrium value. In such circumstances a slow dynamics of the vorticity is described by the equation [9], [6]

$$\mathbf{\Omega}_t = \text{curl} \left[ \text{curl} \left( \frac{\delta \mathcal{H}_*}{\delta \mathbf{\Omega}} \right) \times \frac{\mathbf{\Omega}}{\rho_0(\mathbf{r})} \right], \quad (13)$$

where the Hamiltonian of the vorticity  $\mathcal{H}_*\{\mathbf{\Omega}\}$  is obtained from the quadratic (on  $\delta \mathbf{p} = \mathbf{p} - \mathbf{p}_0$  and  $\delta \rho = \rho - \rho_0$ ) part  $\mathcal{H}^{(2)}\{\delta \rho, \delta \mathbf{p}\}$  of the total Hamiltonian after fixing the potential  $\varphi$  by condition of zero density variation  $\delta \mathcal{H}^{(2)}/\delta \varphi = 0$ :

$$\mathcal{H}_*\{\mathbf{\Omega}\} = \mathcal{H}^{(2)}\{0, \text{curl}^{-1} \mathbf{\Omega} + \nabla \varphi\} \Big|_{\delta \mathcal{H}^{(2)}/\delta \varphi = 0} \quad (14)$$

It should be noted that although the equilibrium solution must be independent on time, the Hamiltonian itself may contain an explicit dependence on  $t$ . It is the case in the space-time with the Kasner metrics (1). In such a Universe the homogeneous equilibrium solutions  $\rho = \rho_0 = \text{const}$ ,  $\mathbf{p} = \mathbf{p}_0 = \text{const}$  are possible [2], which fact can easily be seen from equations of motion (3) and (6).

Now we pass to concrete calculations. We will use the Kasner metrics (1) and the ultra-relativistic equation of state

$$\varepsilon(n) \sim n^{4/3}, \quad (15)$$

since  $n \rightarrow \infty$  when  $t \rightarrow 0$  [2]. For convenience, we change definition of the exponents:  $3\lambda_\alpha = 2\mu_\alpha$  and perform the substitution  $t^{2/3} = \tau$  for the time coordinate in the action  $S$ , and also choose in an appropriate manner scales for spatial coordinates and for relative density  $\rho$ , in order to obtain

$$\mathcal{L}\{\rho, \mathbf{v}\} = -\frac{3}{4} \int \rho^{4/3} \left( 1 - \sum_\alpha \tau^{2\mu_\alpha} (v^\alpha)^2 \right)^{2/3} d\mathbf{r}. \quad (16)$$

The procedure of calculation of the Hamiltonian  $\mathcal{H}\{\rho, \mathbf{p}\}$ , corresponding to this Lagrangian, includes first of all the solution of the relation  $\mathbf{p} = (1/\rho)(\delta \mathcal{L}/\delta \mathbf{v})$ , i.e.

$$p_\alpha = \rho^{1/3} \left( 1 - \sum_\beta \tau^{2\mu_\beta} (v^\beta)^2 \right)^{-\frac{1}{3}} \tau^{2\mu_\alpha} v^\alpha, \quad (17)$$

respectively to  $\mathbf{v}$ , and its substitution to (9). Let us introduce the notations

$$A = \sum_\beta \tau^{2\mu_\beta} (v^\beta)^2, \quad Q = \rho^{-2/3} \sum_\beta \tau^{-2\mu_\beta} p_\beta^2. \quad (18)$$

We define the auxiliary function  $h(Q)$  by the conditions

$$Q^3(1-A)^2 = A^3, \quad h = \sqrt{AQ} + \frac{3}{4}(1-A)^{2/3}. \quad (19)$$

The explicit expression  $h(Q)$  can be found with the help of formula for roots of cubical polynome. Designating for brevity

$$R(Q) = 2^{-1/3} \left( 27 - 2Q^3 + 3\sqrt{3}\sqrt{27 - 4Q^3} \right)^{1/3}, \quad (20)$$

we obtain that

$$h(Q) = \frac{Q}{3} \left( -Q + \frac{Q^2}{R(Q)} + R(Q) \right) + \frac{3}{4} \left( 1 - \frac{Q}{9} \left( -Q + \frac{Q^2}{R(Q)} + R(Q) \right)^2 \right)^{\frac{2}{3}}. \quad (21)$$

We plot the dependence  $h$  versus  $\sqrt{Q}$  in the Fig. 1.

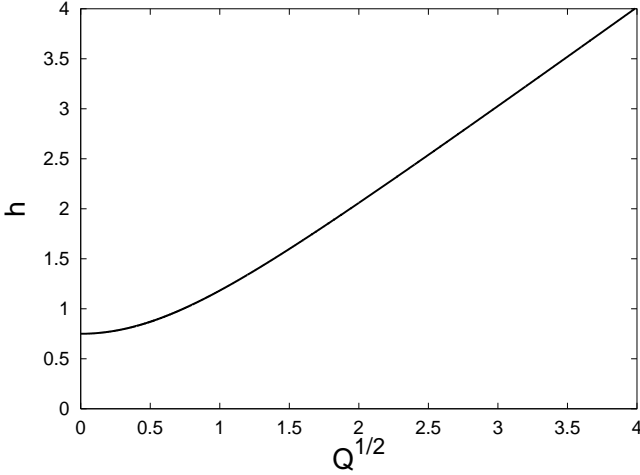


FIG. 1. The dependence  $h$  on  $\sqrt{Q}$ .

At small  $Q$ , the approximate equality is valid

$$h(Q) \approx \frac{3}{4} + \frac{Q}{2}, \quad Q \ll 1, \quad (22)$$

and at  $Q \gg 1$  the following behaviour takes place:

$$h(Q) \approx \sqrt{Q} + \frac{1}{4Q}, \quad Q \gg 1. \quad (23)$$

As the result, the Hamiltonian of the ultra-relativistic fluid in the Kasner space-time has the form

$$\mathcal{H}\{\rho, \mathbf{p}\} = \int \rho^{4/3} h \left( \rho^{-2/3} \sum_\alpha \tau^{-2\mu_\alpha} p_\alpha^2 \right) d\mathbf{r}. \quad (24)$$

For definiteness, we arrange the Kasner exponents in the order  $\mu_1 < \mu_2 < \mu_3$ . The constants  $\mu_\alpha$  satisfy the inequalities

$$-2 < 2\mu_1 < -1, \quad -1 < 2\mu_2 < 1, \quad 1 < 2\mu_3 < 2, \quad (25)$$

following from the conditions (2). Let us suppose  $z$ -component of the mean momentum be different from zero:  $p_{z0} \neq 0$ . We consider here the case when deviations of  $\mathbf{p}$  from the mean value are comparatively small. Then at sufficiently small times the quantity  $Q$  reaches large values, since the order of magnitude of quantities  $\rho$  and  $\mathbf{p}$  is conserved due to the conservation laws for fluid amount and for mean momentum. Asymptotics (23) gives the approximate Hamiltonian in this limit

$$\tilde{\mathcal{H}} \approx \int \left\{ \rho \left( \sum_\alpha \tau^{-2\mu_\alpha} p_\alpha^2 \right)^{\frac{1}{2}} + \frac{\rho^2/4}{\left( \sum_\alpha \tau^{-2\mu_\alpha} p_\alpha^2 \right)} \right\} d\mathbf{r} \quad (26)$$

If we neglect here the quadratic on  $\rho$  term, where the coefficient in front of  $\rho^2$  is small in the limit of small times, then equation of motion for the canonical momentum field does not depend on  $\rho$  at all. Each point of fluid, labeled by a label  $\mathbf{a} = a^\alpha$ , moves almost independently on other points in accordance with the approximate law

$$x^\alpha(\tau, \mathbf{a}) \approx a^\alpha + \int_0^\tau \frac{\tau^{-2\mu_\alpha} p_\alpha(\mathbf{a}) d\tau}{\sqrt{\sum_\beta \tau^{-2\mu_\beta} p_\beta^2(\mathbf{a})}} \approx a^\alpha + \frac{p_\alpha(\mathbf{a}) \tau^{1+\mu_3-2\mu_\alpha}}{|p_z(\mathbf{a})|(1+\mu_3-2\mu_\alpha)}. \quad (27)$$

Deviations of arbitrary constants of motion  $\mathbf{p}(\mathbf{a})$  from the mean value make possible finite-time singularities of the mapping  $\mathbf{r}(\tau, \mathbf{a})$ . Accordingly, quasi-two-dimensional domains of pancake-like shape may arise in the space where the density is much larger than the mean density:  $\rho = \rho(\mathbf{a}) \det \|\partial \mathbf{a} / \partial \mathbf{r}\| \gg \rho_0$ . There, the neglect of the quadratic on  $\rho$  term is no more valid, and actually a redistribution of momenta between fluid elements occurs.

To investigate slow flows on a resting background, it is necessary to obtain the expression for  $\mathcal{H}^{(2)}\{\delta\rho, \mathbf{p}\}$ . Let us expand  $\mathcal{H}\{\rho, \mathbf{p}\}$  to the second order terms on  $\delta\rho(\tau, \mathbf{r}) = \rho(\tau, \mathbf{r}) - \rho_0$  and  $\mathbf{p}(\tau, \mathbf{r})$ . Then using of the expansion (22) gives

$$\mathcal{H}^{(2)} = \frac{\rho_0^{2/3}}{2} \int \sum_\alpha \tau^{-2\mu_\alpha} p_\alpha^2 d\mathbf{r} + \frac{\rho_0^{-2/3}}{3} \int \frac{(\delta\rho)^2}{2} d\mathbf{r}. \quad (28)$$

Expression (28) and equations (12) allow us immediately to write the linearized equations of motion for the spatial Fourier-harmonics of sound waves:

$$-\frac{d\varphi_{\mathbf{k}}}{d\tau} = \frac{1}{3} \rho_0^{-\frac{2}{3}} \rho_{\mathbf{k}}, \quad \frac{d\rho_{\mathbf{k}}}{d\tau} = \rho_0^{\frac{2}{3}} \left( \sum_\alpha \tau^{-2\mu_\alpha} k_\alpha^2 \right) \varphi_{\mathbf{k}}. \quad (29)$$

We can reduce this system to one ordinary linear differential equation

$$\frac{d^2 \varphi_{\mathbf{k}}(\tau)}{d\tau^2} + \frac{1}{3} \left( \sum_{\alpha} \tau^{-2\mu_{\alpha}} k_{\alpha}^2 \right) \varphi_{\mathbf{k}}(\tau) = 0. \quad (30)$$

If  $k_z^2 \neq 0$ , then behaviour of  $\varphi_{\mathbf{k}}(\tau)$  at small times is determined by the exponent  $\mu_3$ . Obviously, all solutions of equation (30) have in general an oscillating character at  $\tau \gtrsim \tau_*(\mathbf{k}) \sim |k_z|^{1/(\mu_3-1)}$ . There is no oscillations at  $\tau \ll \tau_*(\mathbf{k})$ . Among linearly independent solutions there exists one, which tends to zero at  $\tau \rightarrow 0$  as  $\varphi_{\mathbf{k}}^{(1)} \sim \tau$ . Let us note that the corresponding  $\rho_{\mathbf{k}}^{(1)}$  tends to some finite value as  $\tau \rightarrow 0$ . Another solution, linearly independent on first one, is finite in the zero, but its derivative diverges at  $\tau \rightarrow 0$  as  $d\varphi_{\mathbf{k}}^{(2)}/d\tau \sim \tau^{1-2\mu_3}$ , and this fact means unbounded growth of  $\rho_{\mathbf{k}}^{(2)}$  and indicates transition of the system to the highly nonlinear dynamics regime described by the Hamiltonian (26).

For analysis of slow vortical flows, let us write explicitly the Hamiltonian  $\mathcal{H}_*\{\Omega\}$ . For simplicity we put  $\rho_0 = 1$ . In accordance with the definition (14) and the expression (28), we obtain after simple calculations that

$$\mathcal{H}_*\{\Omega\} = \frac{1}{8\pi} \iint d\mathbf{r}_1 d\mathbf{r}_2 \frac{\sum_{\alpha} \tau^{2\mu_{\alpha}} \Omega^{\alpha}(\mathbf{r}_1) \Omega^{\alpha}(\mathbf{r}_2)}{\left( \sum_{\alpha} \tau^{2\mu_{\alpha}} (x_1^{\alpha} - x_2^{\alpha})^2 \right)^{1/2}}. \quad (31)$$

Let us express the frozen-in field of the vorticity through the shape  $\mathbf{R}(\nu, \xi, \tau)$  of vortex lines, (see [8], [9], [10] for details):

$$\Omega(\mathbf{r}, \tau) = \int_{\mathcal{N}} d^2\nu \oint \delta(\mathbf{r} - \mathbf{R}(\nu, \xi, \tau)) \frac{\partial \mathbf{R}}{\partial \xi} d\xi, \quad (32)$$

where  $\nu$  is a vortex line label, which belongs to the 2D manifold  $\mathcal{N}$ ,  $\xi$  is a longitudinal parameter along the line. The dynamics of vortex lines is determined by the variational principle  $\delta S_{\mathbf{R}} = \delta \int \mathcal{L}_{\mathbf{R}} d\tau = 0$ , with the Lagrangian of vortex lines having the following form [6], [8]- [10]:

$$\mathcal{L}_{\mathbf{R}} = \frac{1}{3} \int_{\mathcal{N}} d^2\nu \oint \left( [\mathbf{R}_{\tau} \times \mathbf{R}] \cdot \mathbf{R}_{\xi} \right) d\xi - \mathcal{H}_*\{\Omega\{\mathbf{R}\}\}, \quad (33)$$

where the subscripts  $\tau$  and  $\xi$  mean the corresponding partial derivatives. Performing the deforming substitution

$$\mathbf{R} = (X, Y, Z) = (\tau^{-\mu_1} \tilde{X}, \tau^{-\mu_2} \tilde{Y}, \tau^{-\mu_3} \tilde{Z}), \quad (34)$$

we obtain the Lagrangian for  $\tilde{\mathbf{R}}(\nu, \xi, \tau)$ :

$$\begin{aligned} \mathcal{L}_{\tilde{\mathbf{R}}} &= \frac{1}{3} \int \left( [\tilde{\mathbf{R}}_{\tau} \times \tilde{\mathbf{R}}] \cdot \tilde{\mathbf{R}}_{\xi} \right) d\xi d^2\nu \\ &- \frac{1}{8\pi} \iint \frac{(\tilde{\mathbf{R}}_{\xi_1}(\nu_1, \xi_1) \cdot \tilde{\mathbf{R}}_{\xi_2}(\nu_2, \xi_2))}{|\tilde{\mathbf{R}}(\nu_1, \xi_1) - \tilde{\mathbf{R}}(\nu_2, \xi_2)|} d\xi_1 d^2\nu_1 d\xi_2 d^2\nu_2 \end{aligned}$$

$$\begin{aligned} &- \frac{1}{3\tau} \int \left( \tilde{X}_{\xi} \tilde{Y} \tilde{Z} (\mu_2 - \mu_3) + \tilde{Y}_{\xi} \tilde{Z} \tilde{X} (\mu_3 - \mu_1) \right. \\ &\quad \left. + \tilde{Z}_{\xi} \tilde{X} \tilde{Y} (\mu_1 - \mu_2) \right) d\xi d^2\nu \end{aligned} \quad (35)$$

It is interesting to note that the same Lagrangian for vortex lines corresponds to the usual Eulerian incompressible hydrodynamics in isotropic space with coordinates  $\tilde{\mathbf{r}} = (\tilde{x}, \tilde{y}, \tilde{z}) = (\tau^{\mu_1} x, \tau^{\mu_2} y, \tau^{\mu_3} z)$ , but in the presence of external straining potential velocity field

$$\tilde{\mathbf{v}}_{ext}(\tau, \tilde{\mathbf{r}}) = \tau^{-1} (\mu_1 \tilde{x}, \mu_2 \tilde{y}, \mu_3 \tilde{z}). \quad (36)$$

In conclusion, let us say a few words about hydrodynamics in the homogeneous cosmological models of the other types [2], where the space is not flat. In this case, a quantitative analysis is difficult due to curvature of space and a complicated dependence of metrics on time, but the qualitative result about initial free motion regime for fluid elements remains valid. The reason is that an appropriately defined relative density  $\rho$  is conserved in order of magnitude, as well as covariant components  $p_{\alpha}$  of the momentum field, while contravariant components  $p^{\alpha}$  tend to infinity at  $t \rightarrow 0$ , like in the flat model. This fact means an infinite growth of the quantity  $p_{\alpha} p^{\alpha}$  and approaching of the velocity magnitude  $\sqrt{v_{\alpha} v^{\alpha}}$  to the speed of light, regardless field  $\rho$ . Therefore, a weak initial spatial inhomogeneity in distribution of  $p_{\alpha}(\mathbf{a})$  may result, after some time, in a strong spatial inhomogeneity of  $\rho$ .

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